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Dynamic Modeling and Control of Antilock Braking System for Four Wheel Vehicles

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Abstract: Nonlinear dynamics of longitudinal and lateral traction of ground vehicles has been investigated in this work and later on used these dynamics on usefulness of brake steer system which uses differential brakes for small steering interventions to keep the vehicle on line. A controller design for Antilock braking (ABS) of a four wheel vehicle whose two rear wheels are braking and front wheels are non-braking is proposed. The major part of our work is tuning to the suitable braking force for two rear wheels and minimizing the stopping distance in short time while front wheels are not braking then finding steer of vehicle by applying different forces on rear right and rear left wheels. These two different forces produce rotational torque in vehicle . These are known as differential brakes which are commonly used to steer the vehicle and its lateral position. Stability has been discussed. Both lateral and longitudinal control motion of the vehicle are simulated. The proposed control scheme is successfully tested in MATLAB simulation.

Keywords: Antilock braking system, differential brakes, rear wheels, stability, control, steer

1. Introduction

Antilock Braking System (ABS) is a reliable system for vehicle security; it enables the wheels to keep up contact with the road surface. It prevents the wheels from locking, due to locking of wheel vehicle control is lost and any thing dangerous can happen. This system utilizes the standards of threshold braking. An improved control over vehicle dynamics is possible through ABS as it helps with the sudden locking of wheel on dry and wet surfaces alike.

However, according to research its reaction on snow covered areas is still not great. ABS gives speedier reaction and preferred control in comparison with human reaction time. The primary function of ABS is to determine the slip rate using the likely wheel speed, which is actually acquired by the corresponding speed sensor. To maintain the slip rate at a desirable level

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throughout the ranges, the ECU (Electronic control unit) modifies the brake pressure regulator by realizing the slip rate.A wide range of control techniques for ABS frameworks have been produced. These techniques contrast in their hypothetical premise and execution according to variability in road conditions.
ABS incorporates standard hydraulic stopping mechanism and antilock parts which affect its control properties. Because of the discontinuous and unpredictable relationship between friction and slip ABS control becomes a highly nonlinear control problem. Another hurdle in controlling is the unavailability of the direct measure of linear velocity of wheel so it has to be estimated. Tire and road friction is also not measurable directly or may need specialized sensors. Several control techniques have been used to solve the aforementioned problem.

2. Background and Literature Review

Depending on a large span of operating conditions the demand for automotive safety system is increasing day by day [6, 9, and 12]. A controller for passenger cars was revealed in 1978 by Bosch named as ABS, with the main goal of avoiding wheel-lock, minimizing stopping distance, and strengthen steer ability during braking. There are theoretically two kinds of control procedures being utilized as a part of ABS controllers: (a) based upon controlling acceleration, and (b) based upon controlling slip. Automotive braking systems are mainly used for safety issues while focusing upon active safety [18].
The research related to design control for antilock braking systems via the sliding mode approach for highly uncertain situation is lacking due to input (slip) not being measureable [1]. They [2] implement high demand friction coefficients for estimation of coefficients of a polynomial approximation to u (velocity) vs. slip data. Experimentally when test car drives on a wet road, the aforementioned research fails to provide the slope of the slip curve. Clearly, effectively all four wheels brake, so the non-brake wheel is not available as a reference for speed.
A solution to this problem is to estimate speed using ground-pointing radar, global positioning systems, or vehicle speed observers [3]. Locked rear tires can cause under steer and cause the car to spin [4]. The main purpose of slip control is to control the wheel slip to an optimum value to maximize the tire-road friction of the vehicle braking system in which he used adaptive control method and had stopping distance of 26.8 m[5]. This paper presents the method used to determine the PID values for the anti-lock braking system (ABS) of Malaysian passenger cars in which he find the braking distance of 58 m [5]. An ABS that is resistant to external interference, such as fluctuations in the frictional force between tires and the road surface due to changes in road surface conditions and load, is produced in which he reduced the stopping distance to 15.983 m [6] with SMC. This manuscript presents the development of a PID controller for an Antilock Braking System (ABS) using vehicle longitudinal model [8] only. An ABS that is resistant to external interference, such as fluctuations in the frictional force between tires and the road surface due to changes in road surface conditions and load, is proposed with a stopping distance of 40 m and braking distance of 53.3m.

3. Dynamic Modeling of Vehicles

To plan a controller, a representative model of the framework is required. A vehicle scientific model, which is proper for both speeding up and deceleration, is depicted in this section. This model will be utilized for outline of control laws and PC recreations. In spite of the fact that the model considered here is generally basic, it holds promise for the progression of the framework.
The task is to build an understanding of the non-linear dynamics of longitudinal and lateral traction of the ground vehicles that is later applied to model brake steer system comprising of differential brakes for small steering interventions that would keep the vehicle in a straight path. Differential

brakes are the factor that lesson vehicle's speed as well as produce a yaw maneuver to control the lateral position of the vehicle. The force which significantly contributes in braking and acceleration is the *traction force*. These are frictional forces other than rolling friction of the road and tire.
In the formulation of dynamics three state variables, longitudinal velocity, wheel rotational velocity and wheel slip are considered. First, the differential equations using longitudinal velocity rate of the vehicle and angular velocity rate of the wheel are obtained then simplifying to get differential equations of longitudinal velocity rate and slip
Some well-known longitudinal tire models are presented. The tire models represent the friction coefficients as a function of slip. These are empirical formulae and can be determined from experimental data. An empirical relationship for common tire and friction, hard asphalt is chosen for this work. Analytical tire model has not been discussed here. Steady slip conditions and stability of dynamical system has also been discussed.
In steady slip state wheel slip remains constant and is unassociated from the vehicle speed. Under this condition the vehicle speed gradually drops to zero. Thus the stopping distance and time can be calculated. Since slip rate changes by changing the applied torque, it has implication in the analysis discussed next. In stability analysis the range of applied torque can be decided where the dynamics achieves stability or instability.
The lateral tire dynamics has been discussed. Lateral forces between tire and road produced during turning produce slip angle and camber angle are important in the calculation of lateral tire model. A four wheel vehicle with two rear wheels taken as braking wheels and front wheels without brakes is also discussed. Equations of motion are described taking into account braking forces and other external forces like aerodynamic force, drive line forces and uphill forces etc. Normal load transfer on the wheels during braking (acceleration) known as longitudinal load transfer is formulated and lateral load transfer is also considered in case of turning.
Upon the application of different braking torque on rear wheels a moment generates which steers the vehicle and produces a yaw moment in the vehicle. The model equations of motion are written in first order coupled differential equations. To solve these equations of motion we can use numerical integration methods. In our case the forces aerodynamic forces, drive line forces and uphill gradient are neglected and longitudinal acceleration D_x is calculated for each three wheels individual load distributed at each wheel. The rolling friction for each wheel can be calculated from simple friction law for each wheel, $F_{r_i} = \mu_r F_{z_i}$ Where F_{z_i} is the normal force on each wheel and μ_r is the coefficient of rolling friction ranging from 0.02 to 0.05.

3.1 System Dynamics

The model takes the wheel speed and the vehicle speed as state variable, and it recognizes the torque connected to the wheel as the information variable. The two state factors in this model are related with one-wheel rotational flow and direct vehicle progression. The state conditions are the consequence of the utilization of Newton's law to wheel and vehicle elements.

3.1.1 Single Wheel Dynamics

Let us suppose that a single wheel having an effective radius of R is moving with the rotational velocity of 'ω' with the vehicle speed of longitudinal velocity 'v'. Consider F_{rr} and F_{rl} as the two longitudinal braking forces of the right and left rear wheels of the vehicle when different braking torques are applied. Let front wheel of the vehicle not be involved in braking. Formally we will discuss only single wheel longitudinal dynamics and traction estimation for simplicity, however, later on two-wheel (two axle vehicles) braking model will also be discussed. An illustration of the wheel is shown in the Fig-1

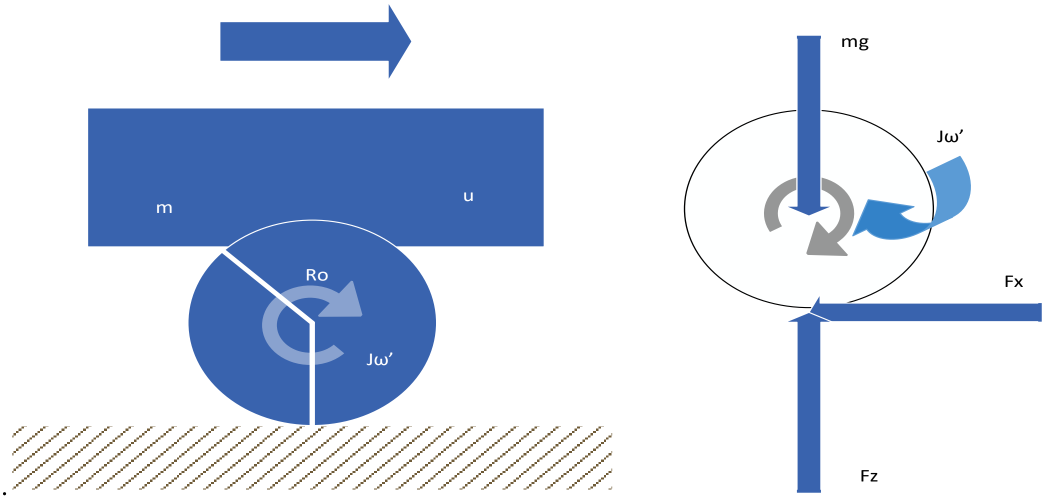

Fig. 1 - Single Wheel braking model and free body diagram

The equation of motion follows by Newton's law as in Eq.(1) ;

$$m\dot{u} = -F \text{.............}(1)$$

Static forces relate to the weight of the vehicle as:

$$F_Z = mg \text{.............}(2)$$

Euler's equation gives:

$$J\dot{\omega} = RF_x - T_b \text{...........}(3)$$

$$F_x = \mu(s)F_z \text{ .... } (4)$$

Where "J" is denoted as the moment of inertia of tire, "R" represents the effective radius "m" represents the mass, g is the gravitational acceleration and $\mu(s)$ is the coefficient of friction estimated by different friction models. It depends upon different road conditions (dry, wet, gravel), speed of vehicle and wheel slip "s". Here we shall find the dependence of $\mu(s)$ on s only; s is the longitudinal wheel slip. Coefficient of friction is a dimensionless quantity which is defined in Eq-5

$$\mu(s) = \frac{u - \omega R}{max(u, \omega R)} \text{.....................}(5)$$

Where "u" is the linear velocity of the vehicle and $R\omega$ is the circumferential speed of the wheel relative to center. The value of $s \in [0,1]$ in case of braking and $s \in [0,-1]$ in case of acceleration For $s = 0$ implies that $u = R\omega \neq 0$ that free rolling with $T_b = 0$ and non-zero initial speed and $s = 1$ for $R\omega = 0$ which is braking lock up.

$$\dot{\omega} = \frac{R}{J}F_x - \frac{T_b}{J}$$
$$\dot{\omega} = \frac{R}{J}\mu(s)mg - \frac{T_b}{J}$$
$$\dot{\omega} = \frac{g}{R}[\frac{mR^2}{J}\mu(s) - \frac{RT_b}{J}]$$
$$\dot{\omega} = \frac{g}{R}[v\mu(s) - \tau_b]................(6) where, v = \frac{mR^2}{J} \; \& \; \tau_b = \frac{RT_b}{J}$$

In these equations 'u' and 'ω' are computed in a complex way. Alternatively we can take 'u' and 's' as dynamic states. Where $u \in R^+$, $s \in I = [0\ 1]$. Now, the slip condition is written as: $S = \frac{u - \omega R}{u}$

3.2 Four Wheels Vehicle Model

Differential brakes are applied on the rear wheels of the vehicle; a steer torque imposes on the vehicle which is given by Eq. 7:

$$M_{BS} = \frac{T}{2}(F_{rr} - F_{rl}) = \frac{T}{2}F_{BS}..........(7) where, F_{BS} = (F_{rr} - F_{rl})$$

Now, 'T' is denoting the width of the vehicles and 'F_{rr}' and 'F_{rl}' are the respective right and left longitudinal tire forces. 'M_{BS}' Produces yaw rate 'r' which in turn affects the front and rear side slip angles α_1 & α_2 respectively as:

$$\alpha_1 = \tan^{-1}(\frac{v - cr}{u}), \alpha_2 = \tan^{-1}(\frac{v + br}{u})$$

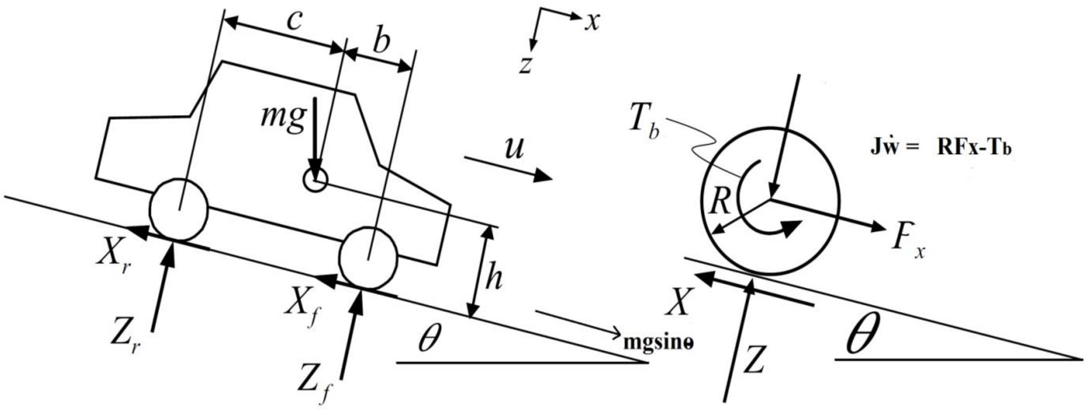

Fig. 2 - A four wheel model on inclined plane

The modeled equations are written in first order coupled differential equations. To solve these equations of motion we can use numerical integration methods.

$$\dot{u} = \frac{1}{M}[-\mu_{rr}(s)Z_{rr} - \mu_{rl}(s)Z_{rl} - \mu_r Z_{rr} - \mu_r Z_{rl} - \mu_r Z_{fl} - \mu_r Z_{fr} - D_a - F_d - Mgsin\theta......(8)$$

$$\dot{v} = \frac{F_{yrl}}{M} - \frac{F_{yrr}}{M} - \frac{F_{yfl}}{M} - \frac{F_{yfr}}{M} - ru.......................(9)$$

$$\dot{r} = \frac{-bF_{yrl} - bF_{yrr} - cF_{yfl} - cF_{yfr} + M_{BS}}{I_Z}...........(10)$$

$$\dot{s}_r = \frac{g}{u}[\frac{-\mu_{rr}(s)_{rr}R}{u} - \frac{\mu_{rr}(s)Z_{rr}R^2}{Jg} + \frac{T_{brr}}{Jg}].......(11)$$

$$\dot{s}_r = \frac{g}{u}[\frac{-\mu_{rl}(s)_{lr}R}{u} - \frac{\mu_{rl}(s)Z_{rl}R^2}{Jg} + \frac{T_{brl}}{Jg}].........(12)$$

$$\dot{X} = V_x..........(13)$$

$$\dot{Y} = V_y...........(14)$$

$$\dot{\psi} = r.............(15)$$

4. Frictional Model

4.1 Longitudinal Friction Models

The scope of this research will include two renowned empirical models

1) Burckhardt Model

Mathematical form of this model is: $\mu(s) = c_1\left(1 - e^{-c_2 s}\right) - c_3 s.........(16)$

Where 'c_1', 'c_2' and 'c_3' are the constants that can be obtained experimentally. The modified form of this formula, with an additional term '$e^{-c_2 s}$' describes the dependence of coefficient of friction on the velocity of the vehicle as;

$$\mu(s) = \left(c_1\left(1 - e^{-c_2 s}\right) - c_3 s\right)e^{-c_2 s}............(17)$$

Neglecting the term '$e^{-c_2 s}$' and assigning the values to constants, $c_1 = 1.18, c_2 = 10, \& c_3 = 0.5$, a simple relation is obtained. $\mu(s) = 1.18\left(1 - e^{-c_2 s}\right) - 0.5$ Where $s \in I$. The curve of '$\mu(s)$' is uni-model and has a peak value $\mu_{\max} = 0.972$ at $s = 0.316$ in μ-s graph.

2) Piecewise linear Model

This model estimates the Burckhardt model's μ-s curve as a piecewise linear function. This linear approximation of the coefficient of friction from Burckhardt model is shown in Fig-3.

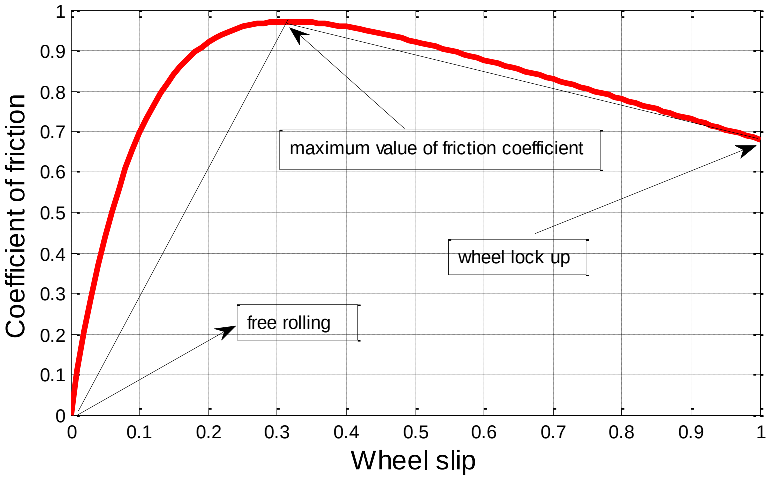

Fig. 3 - Piece wise linear model from slip dynamics

4.2 Control Law

ABS Control

The incredibly complex relationship between the segments and parameters of an ABS control results in a highly nonlinear control problem. A wide array of research has been performed in this area. A wide range of ABS control techniques have been developed and research into improved control strategies is ongoing. Most of these methods require a frame model, but some of them are unlikely to perform compellingly under varying road conditions progression. ABS brake control is very difficult. Below are some hurdles to overcome with brake control.

a) For optimal performance, the controller should operate at an unstable equilibrium point.

b) Different road conditions can lead to wide variations in the maximum braking torque.

c) The tire slip estimation flag, which is essential for control execution, is very dubious and noisy.

d) On wet roads, tire bobbing causes normal and rapid fluctuations in tire slip ratio.

e) The coefficient of friction of the brake pads is also variable.

f) Brake frames contain a transport delay that limits the data transmission of control frames.

The ABS comprises of an ordinary hydraulic slowing mechanism in addition to antilock segments which influence the control qualities of the ABS. ABS control is an exceedingly nonlinear control issue because of variably unpredictable connection amongst friction and slip. Another obstacle in this control issue is that the linear velocity of the wheel isn't specifically quantifiable and it must be assessed.

5. Results and Discussion

Control issues that are emerging in a wide range of designing fields are described by basic uncertain conditions and nonlinearities. Because of nonlinearities and certain progressions the framework parameters are uncertain or hard to figure so control exactness is lost. Our problem is to design such an ideal mathematical modeling which can be employed for easy control. We consider all the possible parameters which are faced practically during travelling. The vehicle parameter like lateral and longitudinal dynamics are considered in our modeling. When vehicle is in the turning phase the load is transferred to the sides. We also modeled that lateral load transfer and choose dry asphalt during coding of our system.

Steer ability while braking is crucial for minor course adjustments as well as for the likelihood of directing around a hindrance so steer is focused . Exactly when the vehicle is under rapid braking move, for a little associated braking torque, the wheel speed starts to decrease and the wheel slip starts to increase from zero. If the slip is under 0.2, then with the rising of slip, frictional coefficient furthermore constricts which is achieving the rising of frictional power. In this manner wheel speed increases, realizing the decay of slip. All these situations are considered in mathematical modeling and control design.

5.1 Simulation & Results

First of all the differential equations are developed in s-block. The dynamic equations of our plant are modelled in SIMULINK followed by control design developed in s-block. Subsequently controller is then modeled in SIMULINK. The stopping criterion of slip is taken 0.2.

The equations are modeled in s-function with following vehicle parameters as shown in Table 1

Table 1: Detail of Parameters

S.NO	***Parameters***	***symbols***	***values***
1.	Mass of vehicle	M	1543.6 kg
2.	Gravitational acceleration	g	9.81 m/s^2
3.	Momentum of inertia	J	0.1953 kgm^2
4.	Radius of wheel	R	0.18
5.	Coefficient of friction	μ	0.02 kgm^2
6.	Moment of inertia along z-axis	I_z	1.160397e+003
7.	Distance of front wheel of C.G	c	2.415
8.	Distance of rear wheel of C.G	b	0.26
9.	Length of axle	L	a+b =2.675
10.	Angular velocity of wheel	ω	1.34 rad/s
11.	Height of vehicle from center of gravity	h	1.0744
12.	Height of point of rotation from ground during turning or load transferring	h_r	1.0
13.	coefficient of cornering stiffness	$C_{\alpha 1}$	50.0
14.	coefficient of cornering stiffness	$C_{\alpha 2}$	50.0
15.	Cambering Coefficient	C_γ	0.0
16.	Camber Angle	γ	0.0349
17.	Aerodynamic drag	D_a	0.0
18.	Steer angle	θ	0.0
19.	Derive line force	F_d	0
20.	Braking torque for rear right wheel	$T_{b\,r\,r}$	1500 Nm
21.	Braking torque for rear left wheel	$T_{b\,r\,l}$	1500 Nm

The longitudinal velocity is shown in Fig-4. It becomes to zero in a very short time, t = 5 sec.
The well tracking of angular velocity of right & left wheel is shown in Fig-4. It can be seen that the angular velocity
of right & left wheel are well track.

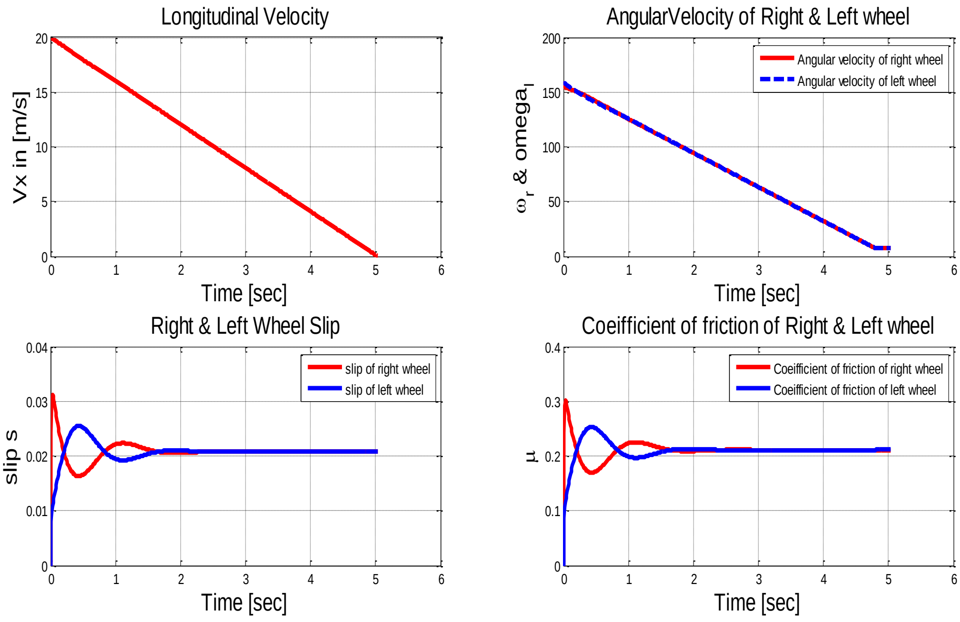

Fig. 4 - Response of angular velocity of right & left wheel

The slip of right and left wheel becomes zero in suitable time and the tracking of the coefficient of friction of right and left wheel is shown above figure with satisfactory results as shown in Fig. 5.

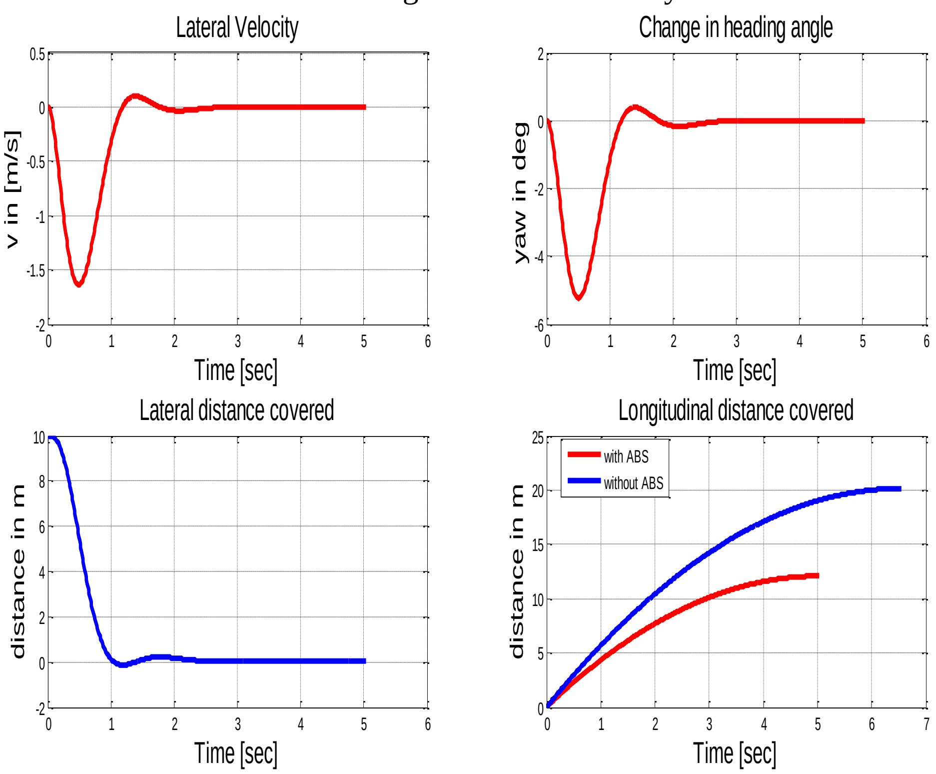

Fig. 5 - Response of lateral distance

It can be seen that the lateral velocity is zero when the vehicle is moving forward and when the braking torque is applied then it decreases to -1.6 m/sec. Afterwards it then settles to zero in a very short period of time, t= 2.6 s This indicates that the proposed scheme gives very satisfactory result. The heading angle of the vehicle is shown in figure 5. The yaw is changed from zero to -5.2^0 after applying braking torque and settled in a time of 2.6 seconds. The lateral distance is show in Fig. 5. The lateral distance covered by the vehicle is settled in a suitable time by using the proposed control scheme. The stopping distance of the vehicle is shown in Fig.5. The plot shows that the vehicle is stopped with small longitudinal distance in very short time which is our ideal required position.

6. Conclusion

An ABS system based on differential braking has been demonstrated in simulation. A vehicle model which is efficient in terms of mathematical modeling for control design and simulation is used. Longitudinal and lateral dynamics are considered in this work. Controller is developed using linear design methods, and represent example of traditional PID. The simulation model shows the effect of brakes. The response times are within useful bounds in terms of providing steering intervention for maneuvers to avoid road departure. It is proved that after applying slightly different braking torques vehicle travels in stable condition.

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